A Qualitative Theory for Shape Representation and Matching

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Abstract

A new method for qualitative shape recognition and matching of objects in designs is presented in this paper. The approach consists of a reference-points information approach of the qualitative description of shapes considering qualitatively their angles, relative side length, concavities and convexities, and types of curvatures of the boundary. The shapes recognised are regular and non-regular closed polygons that can have curve segments and curvilinear shapes. Moreover the shapes can contain holes. To describe shapes with holes, topological and qualitative spatial orientation aspects are considered in order to relate the hole with its container. Each object is described by a string containing its qualitative distinguished features (symbolic representation), which is used to match the object against others. The paper also describes how this method can be used in industrial design by explaining an application. Given a file with different objects (representing tiles) and a vectorial image design of a ceramic tile mosaic border, the application recognises which tile in the file belongs to the border design and indicates its position and rotational angle to place the tile in the correct position of the border design. This qualitative method provides several advantages over traditional quantitative representations. The main advantages are the reduction of computational costs and the managing of uncertainty (two manufactured tiles are not geometrically identical, but they represent the same tile in the design).

Introduction

Our environment is full of objects which can be described in terms of their shape. The shape of an object is the description of the properties of the boundary of the object. The boundary of the object is described by a set of points. A single point has neither dimension nor shape, but a one-dimensional curve has a shape that can be described.

A purely quantitative representation of figures consists of a set of mathematical functions of space coordinates. For instance a circumference can be described by the following mathematical function:

$$x^2+y^2=r^2$$

For more complex shapes, it is generally difficult to find a numerical function for the curve or surface describing the boundary of the figure. Piecewise interpolation methods are often used as a simplification.

This means that the object to be described is approximated as consisting of many small parts, for instance straight lines or flat surfaces, for which it is possible to find numerical functions. The set of functions then make up the quantitative description of the shape of the object. An alternative quantitative representation is to approximate the shape of the object by the pixels it occupies. Depending on the resolution, this gives a more or a less coarse result, since some pixels may be only partially filled. Furthermore, the description of the shape may be quite different if it is rotated or translated within the grid.

In the artificial vision field it is necessary a high computational cost for image processing. Moreover, objects recognition from image processing is an unsolved problem, i.e. it is not possible to distinguish the same chair from different points of view or partially hidden by using quantitative image processing.

Therefore, the definition and use of a theory for qualitative description of shape is important in the vision recognition field. The use of a qualitative theory for shape description and recognition will increase the efficiency in vision recognition because the recognition of a shape or an environment will be carried out by looking only for the distinguished features and not analysing each pixel of the image.

A purely qualitative representation may describe shapes by linguistic terms, such as “round”, “straight” and so on. Figure 1 shows an example of a quantitative and a qualitative representation of a circumference.

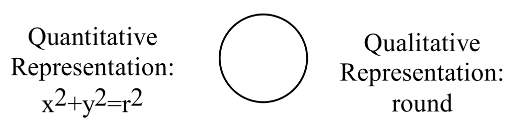


Figure 1. Examples of quantitative and qualitative representation of a circumference.

Most of the qualitative approaches to shape description can be classified as follows:

- Axial representations: these approaches are based on a description of the axes of an object, describing the shape qualitatively by reducing it to a “skeleton” or “axis”. The “axis” is a planar arc reflecting some global or local symmetry or regularity within the shape. The shape can be generated from the axis by moving a geometric figure (named “generator”) along the axis and sweeping out the boundary of the shape. The generator is a constant shape and keeps a specified point (i.e. its centre) but can change its size and its inclination with respect to the axis. Some works inside this group correspond to (Leyton, 1988; Brady, 1983).

- Primitive-based representations: in these approaches complex objects are described as combinations of more primitive and simple objects. Here we can distinguish two schemes:
  - Generalized cylinder and geon-based representations, which describe an object as a set of primitives plus a set of spatial connectivity relations among them (Biederman, 1987; Flynn & Jain, 1991).
  - Constructive representations, which describe an object as the Boolean combination of primitive point sets (Requicha, 1980; Brisson, 1989; Ferrucci & Paoluzzi, 1991).
- Reference points- and Projection-based representation: in these approaches different aspects of the shape of an object are represented by either looking at it from different angles or by projecting it onto different axes (Jungert, 1994; Schlieder, 1996; Freeman & Chakravarty, 1980; Chen & Freeman, 1990; Park & Gero, 1999; Damski & Gero, 1996).
- Topology and logic-based representations: these approaches rely on topology and/or logics representing shapes (Cohn, 1995; Randell & Cui & Cohn, 1992; Clementini & Di Felice, 1997).
- Cover-based representations: in these approaches the shape of an object is described by covering it with simple figures, as rectangles and spheres (Del Pobil & Serna, 1995).

The theory proposed in this article can be classified as an reference points based representation due to the fact that the theory uses the vertices (reference points) of the objects to give its description, and it does not segment complex shapes in primitive shapes, otherwise it gives a unique and complete description of each shape. This theory has been developed to be applied in an application whose goal is the recognition of tiles for the automatic assembling of ceramic mosaic borders by a robot arm. For this application a qualitative approach is the most suitable one because there are not two manufactured tiles exactly identical, and working qualitatively we can manage the uncertainty. The application has two entries, the vectorial design of a mosaic border and a file which simulates the result of a vision system. This file consists on the reference points of the image of real tiles captured by the vision system, which will be extracted in a qualitative way. The file contains information about the reference points and if they belong to a straight or a curve segment. This application requires a new complete method to be able of recognising objects with holes, straight and curve segments in the same model, including too information about the colour and the area of the objects to recognise. The method developed in this approach has been inspired by some of the approaches described above, and it represents a complete method for the application developed.

The Reference-Points Information Approach to Qualitative Shape Representation and Matching

Shape description using reference-points information will have to make use of some reference points. As reference points we understand these points which completely specify the boundary. For polygonal boundaries we have chosen the vertices as reference points. For circular shapes and curvilinear segments in a shape we have chosen three points: the starting and the end point and the point of the curve and its point of maximum curvature.

The qualitative description of a reference point, named *j*, is determined using the previous reference point, named *i*, and following reference point, named *k*. This description is given by a set of three elements (triple) which can differ if these elements are from straight segments of cruvilinear ones:

- In the case of straight segments the triple is <Aj,Cj,Lj> where *Aj* means the angle for the reference point *j*, *Cj* means the type of convexity of point *j* and *Lj* means the relative length of the edges associated to reference point *j* (edge formed by vertices *i* and *j* versus edge formed by vertices *j* and *k*), where:

  Aj ∈ {right-angled, acute, obtuse};

  Cj ∈ {convex, concave} and

  Lj belongs to LRS, where LRS = {smaller, equal, bigger}.
- In the case of curvilinear segments the triple <Curve,Cj,TCj> where the symbol *Curve* means that the node in the description string is describing a curve , Cj means the type of convexity of point j and TCj means the type of curvature of the curve associated to the point j, where:

  Cj ∈ {convex, concave} and

  TCj ∈ {plane, semicircle, acute}

To describe the objects with holes the topological concept of Completely Inside Inverse (CIi) (Isli & Museors et al., 2000) , due to the fact that the hole is always Completely Inside (CI) the boundary of the object in the case of tiles. The cardinal reference system by Frank (Frank, 1991) is used in order to relate the position of the hole inside the object.

As the colour is a relevant characteristic in the case of mosaic tiles, the colour of the shape is stored as RGB colour and then in the matching process the colour is considered qualitatively using the Delta E distance between colours.

Moreover, the size of the tiles is also a relevant feature therefore this feature is also considered in a qualitative way.

The Qualitative Shape Theory for Polygonal Objects without holes

The central idea of the qualitative shape representation consists in given three reference points *i*, *j*, *k*, which are consecutive, the qualitative description of the reference point *j* is determined by positioning an oriented line from the point *i* to the point *k* as figure 2 shows. In figure 2 *i* is the vertex 1, *j* is the vertex 2 and *k* is the vertex 3. In this figure the oriented line is placed from 1 to 3.

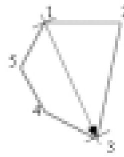


Figure 2. Example of a shape figure in which we are determining the qualitative description of vertex 2 using vertex 1 and 3 by placing and oriented line between them.

Determining the Convexity

The convexity of the point *j* is determined as follows: if the reference point *j* remains on the left of the oriented line from *i* to *k* then the point *j* is a convex vertex. Otherwise if the point *j* remains on the right of the oriented line from *i* to *k* then the point *j* is concave. As a vertex appears when the orientation of the edge changes then it is not possible that the reference point *j* remains exactly over the oriented line from *i* to *k*. Formally, if *Vj* means *vertex j,* and *wrt* means the relation of the vertex j with respect to the oriented line from vertex i to vertex k, we can formulate:

If Vj wrt ViVk $\in$ *left then Vj is convex.*
If Vj wrt ViVk $\in$ *right then Vj in concave.*

Determining the Angle

The qualitative description of an angle is determined using a new concept and some topological concepts as boundary, interior and exterior of an entity. The new concept consist in given the two reference points joined by the oriented line, *i* and *k*, we place a circle of diameter *ik* between these two reference points. Moreover, in order to understand how the angle of a vertex is determined we need to give the definitions of the topological concepts used.

Definition 1. The boundary of an entity h, called δh is defined as:

We consider the boundary of a point-like entity to be always empty.

The boundary of a linear entity is the empty set in the case of a circular line, or the 2 distinct endpoints otherwise.

The boundary of an area is the circular line consisting of all the accumulation points of the area.

Definition 2. The interior of an entity h, called h^o is defined as $h^o = h - \delta h$.

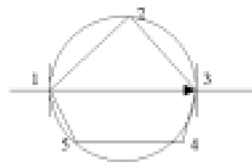
Definition 3. The exterior of an entity h, called h- is defined as $h- = \Re^2 - h$, where $\Re^2$ denotes the 2D Euclidian Space.

Therefore, the angle is determined as follows; if the reference point *j* remains exactly in the boundary of the circle of diameter *ik*, then the vertex *j* is right-angled. If *j* remains in the exterior of the circle then *j* is acute. And if *j* remains in the interior of the circle then the vertex *j* is obtuse. Formally, if the circle with a diameter of ViVk is denoted as Cik, then the angle of the Vertex *j* (Vj) is calculated using the following algorithm:

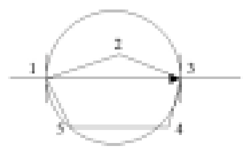
If $Vj \cap \delta Cik \neq \emptyset$ *then Vj is right- angled,*
Else if $Vj \cap Cik^o \neq \emptyset$ *then Vj is obtuse*
Otherwise Vj is acute.

The part of the "otherwise" of the above algorithm occurs when $Vj \cap Cik- \neq \emptyset$.

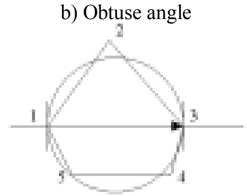
Next figure shows a graphical example for each of these cases (figure 3).



a) Right-angled angle



b) Obtuse angle



c) Acute Angle

Figure 3. Examples of determination of the angle of vertex 2, using an oriented line from vertices 1 and 3and a circle of diameter vertex1vertex3; a) for a right – angled angle; b) for an obtuse angle and c) for an acute angle.

Determining the Length

To determine the relative length of each edge of a rectilinear segment between three contiguous vertices (relative length of edges between the edge from vertex i to vertex j and the edge from vertex j to vertex k) a new length model has been developed which has been inspired in the model by (Hernández & Clementini & Di Felice, 1995) and (Escrig & Toledo, 1998).

The length model developed compares lengths of two consecutive edges of the object. As we compare length at least two lengths are available, and as a result we find that one length is bigger, smaller than or equal to the other.

Therefore the reference system named Length Reference System (LRS) is defined by a set of qualitative lengths labels. Thus, we define the LRS as:

LRS={smaller, equal, bigger}.

The length calculated in the reference point j is the length of the edge from the point i to the point j compared with the length of the edge from the point j to k, using this LRS. Therefore it is inferred as:

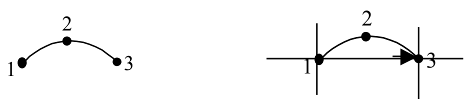
- First the length of each edge is calculate using the Euclidean distance d between two points: $D(Vi,Vj)=((Xvj-Xvi)^2+(Yvj-Yvi)^2)^{1/2}$
- Then, both lengths are compared and the corresponding label of the LRS is assigned as the value of the relative length to the vertex *j*.

The use of the qualitative distance is enough in the application to distinguish shapes, therefore we do not need to use the Euclidean distance anymore.

The Qualitative Shape Theory for Objects with Curves

For describing an object with curves we follow the next steps:

1. First of all the symbol *curve* is fixed to indicate that the next node in the qualitative description of the object corresponds to the description of a curve.
2. To describe qualitatively a curve, 3 points are used: the initial and final points of the curve and the point of maximum curvature of the curve (as depicted in figure 4a), which are obviously consecutive points. The description, however, is associated only to the node of maximum curvature.



a) b)

Figure 4 a) The 3 points considered for the description of a curve and b) the placement of the oriented line between them.

The central idea of the qualitative shape representation for curves consists of, given the three reference points, *i*, *j*, *k*, of the curve which are consecutive, the qualitative description of the reference point *j* (the one of maximum curvature) is determined by positioning an oriented line from the points *i* (the previous point) to the point *k* (the following point) as figure 4b) shows. In figure 4b) the point 1 is point *i*, the point number 2 is the point *j* and the point number 3 in the point *k*.

Determining the Convexity

The convexity (Cj) of the point *j* is determined by the oriented line from *i* to *k* as follows: if the reference point *j* remains on the left of the oriented line from *i* to *k* then the point *j* is a convex vertex. Otherwise if the point *j* remains on the right of the oriented line from *i* to *k* then the point *j* is concave. As *j* is the point of maximum curvature in a curve segment from *i* to *k,* then it is not possible that the reference point *j* remains exactly over the oriented line from *i* to *k*. Formally, if Vj means vertex *j* (reference point which belongs to the one of the maximum curvature), and *wrt* means the relation of the vertex j with respect to the oriented line from vertex i to vertex k, we can formulate:

If Vj wrt ViVk $\in$ *left then Vj is convex.*
If Vj wrt ViVk $\in$ *right then Vj is concave*

Determining the Type of Curvature

The type of curvature (TCj) of the point j is determined by calculating two distances and comparing them (figure 5). For calculating both distance the center point of the line between *i* and *k* is calculated, named point *ik* (Pik). The first distance (da) calculated is the distance between *i* and the new point, Pik, and the second distance (db) considered is the one between the point *j* and the new point (Pik). Then comparing both distances TCj is determined as follows:

If da<db → TCj = acute
If da=db → TCj = semicircle
If da>db → TCj = plane

Figure 6 shows examples of the 3 possible cases.

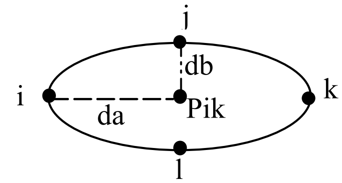


Figure 5. Distances calculated for determining TC_2.

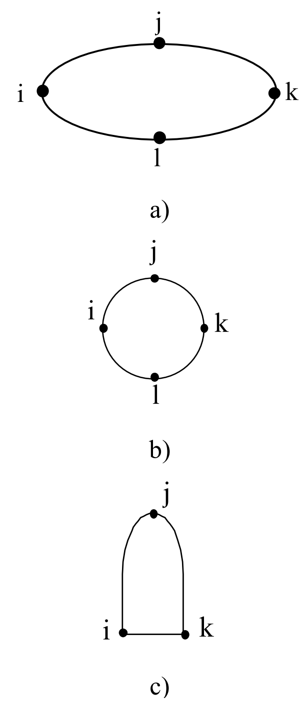


Figure 6. a) Point j has a TCj = plane, b) TCj = semicircle, and c) TCj = acute.

The Qualitative Shape Theory for Objects with holes

For describing an object with holes we follow the next steps:

1. The qualitative shape description of the exterior boundary of the object (container) is constructed following the steps described in previous sections.
2. Then the qualitative shape description of the boundary of each hole is constructed.
3. Each hole and the container are related by adding two types of information:

3.1 The topological relation between the container and each hole is fixed. The holes in the case of tiles are always Completely Inside Inverse (CIi defined in (Isli & Museros et al., 2000) of the container. The CIi is the inverse of the topological relation Completely Inside (CI), which is defined using the formal definition of the "*in*" relation as:

(h1, in, h2) ↔ h1 ∩ h2 = h1 ∧ h°1 ∩ h°2 ≠ ∅.

Given that (h1, in, h2) holds, the following algorithm distinguishes between the completely-inside, the touching-from-inside, and the equal relations:

if (h2, in, h1) then (h1, equal, h2)
else if h1 ∩ δh2 ≠ ∅ then (h1, touching-from-inside, h2)
else (h1, completely-inside, h2)

Therefore, CI_i is defined as:

(h1, completely-insidei, h2) ↔ (h2, completely-inside, h1)

3.2 The orientation of each hole inside the container is determined (this is necessary because we can have objects with a hole which the boundaries of containers are equal and boundaries of the holes too, but the hole is placed in other position of the container and then they are not the same object). The orientation is fixed using Frank's Cardinal Reference System (CRF), which divides space into eight or more cones (which allows working with different levels of granularity) as figure 7 shows. The CRF is defined by placing its origin into the centroid calculated with the definition of the centroid of a close non regular polygon given in (Steger, 1996). In the case of curvilinear shapes or shapes which contain curve segments, these shapes are approximated to polygonal shapes which vertices are the reference points considered for the qualitative description of the shape and these vertices are joined by rectilinear segments. (Steger, 1996) calculates the centroid ($\alpha_{1,0}$ is the x coordinate and $\alpha_{0,1}$ is the y coordinate) in basis of the area (α) as:

$$\alpha = \frac{1}{2}\sum_{i=1}^{n} x_{i-1}y_i - x_i y_{i-1}$$

$$\alpha_{1,0} = \frac{1}{6\alpha}\sum_{i=1}^{n}(x_{i-1}y_i - x_i y_{i-1})(x_{i-1} + x_j)$$

$$\alpha_{0,1} = \frac{1}{6\alpha}\sum_{i=1}^{n}(x_{i-1}y_i - x_i y_{i-1})(y_{i-1} + y_j)$$

We call center (C) to the orientation occurred when the hole is placed environ the centroid, and all orientations hold.

When several orientations hold for a given hole, then the orientation is fixed to a set of all the orientations (figure 7).

Then once the CRF is placed in the object, the orientation of the hole with respect to the object is calculated. For instance, figure 7 calculates the orientation of the hole with respect to the container, obtaining that the hole is [NE,E,SE] oriented inside the container.

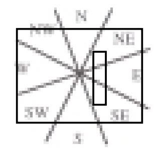


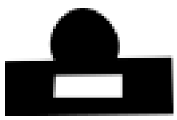
Figure 7. Example of the Orientation Calculation of a hole with respect to its container.

The Complete Description of a Shape

Given a shape its complete description is defined with the following tuple: [holes_type, curves_type, [Colour, [A1,C1,L1 | Curve, C1, TC1]…[An,Cn,Ln | Curve, Cn, TCn]],(CIi,Orientation,[curves_type,[AH1,CH1,LH1|Curve, CH1,TCH1]…[AHj,CHj,LHj |Curve,CHj,TCHj]])^m], where n is the number of vertices (reference points) of the container and j is the number of vertices of the holes (reference points). The holes_type belongs to the set [without-holes, with-holes], the curves_type belongs to the set [without-curves, with-curves, only-curves], both symbols are introduced to speed up the matching process. Colour is the RGB colour of the piece described by a triple as the set [R,G,B] for the Red, Green and Blue coordinates. Each set , [A1,C1,L1 | Curve, C1, TC1] represents a node of the qualitative description which can be the description of a vertex of a rectilinear segment, and then A1,…,An, C1,…,Cn and L1,…,Ln are the qualitative angle, convexity type, and relative length of the vertices of rectilinear segments of the container respectively, or (represented by the symbol |) it can represent a node of the qualitative description of a curvilinear segment and then it is formed by the symbol Curve to indicate that it is a curvilinear segment. C1, …, Cn, and TC1, …, TCn are the qualitative description of the convexity type and curvature type respectively. The same happens with the container of the hole: AH1,..AHj, CH1,…,CHj and LH1,…LHj, are the qualitative angle, convexity type, and relative length of the vertices of rectilinear segments of the hole and CH1,…, CHj, and TCH1,…, TCHj are the convexity type and curvature type of the curvilinear segments of the hole. The string CIi, Orientation, [[AH1,CH1,LH1]… [AHj,CHj,LHj]] is repeated for each hole inside the container. CIi is the topological relation relating the hole with its container. Finally Orientation is the set of orientation relations given by the CRF in order to give the orientation of the hole in the container.

Therefore, in order to describe completely a shape, first we have to repeat the process of giving the qualitative description of each vertex to describe the boundary of the container and the boundary of each hole (if they appear). Then colour is stored as RGB coordinates. The orientation relations between the container and each hole is calculated using the CRF. And the final set (string) with the characteristics of the shape is constructed.

Figure 8 shows an example of a shape with a hole, rectilinear and curvilinear segments and its qualitative shape description, formally named QualShape(S), being S the reference to the object described.



QualShape(S)=[with-holes, with-curves, [[0,0,0], [right-angle,convex,bigger], [curve, convex, acute], [right-angle, convex,bigger], [right-angle, convex, smaller], [right-angle, convex,bigger]],CIi,C,[[right-angle, convex, smaller], [right-angle, convex, bigger],[right-angle, convex, smaller], [right-angle, convex, bigger]]].

Figure 8. Example of a black (RGB = 0R,0G,0B) shape with a hole, curvilinear and rectilinear segments.

The Matching Process

The matching process is made as follows, first the qualitative description of the object taken as reference is constructed as defined in previous sections, and then the qualitative description of the other object to match is constructed up to the description of the container, it means that the holes are not yet described. With this two strings we compare if both are of the same type (with or without holes), same colour and the containers are equal. For comparing the colour qualitatively, as the colour in tiles will be always solid colours, the Delta E distance between colours is used. The Delta E distance using RGB colour systems is calculated as:

Given two colours in RGB, named C1 determined by (R1,G1,B1) and C2 determined by (R2,G2,B2), then the Delta E distance between colours is calculated as the Euclidian distance between the RGB coordinates of each colour as:

Delta_E(C1,C2)=((R1-R2)2+(G1-G2)2+(B1-B2)2)1/2

If the Delta_E is less than 0,2 it is because an experimented human eye in the recognising of colours field cannot differentiate between the two colours.

Due to implementation reasons the vertices of containers and holes of each shape are numbered in a counter clockwise way, being the first vertex (number 1) the uppermost (left) vertex of the shape.

To compare the containers the algorithm ComparingVertices is applied, which is a cyclical ordering matching algorithm which given two set of vertices, returns if both strings are equal cyclically and the vertex in the second object which corresponds to the vertex number 1 in the first one. If both sets are not equal the vertex in the second set is not found, therefore a –1 value is assigned.

Algorithm ComparingVertices (INPUTS: SetVertices1, SetVertices2, OUTPUTS: vertex02, equal){

```
 N=Calculus size SetVertices1
 M=Calculus size SetVertices2
 If N == M then {
          //Both sets have the same number of vertices
          For (I=0;I<N-1;I++) {
             For (J=0;J<N-1;J++){ //cyclic comparison
          //Compare Vertex1(0) of SetVertices1 with Vertex2(j) of
          //setVertices2
             If Vertex1(0) == Vertex2(j) {
                   Num=0 //Init a counter
                   For (K=1;K<N-1;K++){
                     If (Vertex1(K)==Vertex2(J+1%N)) then {
                        NUM++;}
                   If (NUM==N){
                        Return equal=true;
                      Return vertex02= j;
                        Break }
                   } //For K
                } // If Vertex1(0) ==Vertex2(j)
              } //For J.
          } //For I
          If (NUM<>N) {
            Return equal=false;
          Return vertex02= -1;}
 } //If N==M
else {
 Return false;}
} //End Algorithm
```

If the objects have no holes the process finishes here.

This way to start the matching process is motivated due to the objects with holes that are found rotated with respect to the reference object to compare will describe the holes in other orientation to the one given to the reference object being both the same object. Then once we obtain that both objects are equal up to the container, and both contains holes, the string describing the holes of the second object (not the reference object already completely described) is constructed by following next steps:

4. Each hole in the object is numbered as being the vertex number 1 the one closest to the vertex which corresponds to the vertex 1 in the reference object, calculated when the cyclic comparison has been made.
5. Include the string CIi in the qualitative description of the object for the first hole and calculate the orientation of the first hole with respect to the container placing the NW of the RS oriented to the vertex which corresponds to the vertex 1 in the reference object, and include it in the qualitative description of the object.
6. Calculate the qualitative description of the boundary of the first hole ([[AH1,CH1,LH1]…[AHj,CHj,LHj]]) and include this description in the qualitative description of the object.
7. Repeat steps 2,3, and 4 for each hole inside the object.

Once the qualitative description of the second object is completed, then first we compare the number of holes, if both objects have the same number of holes we continue comparing, and we compare each hole of the reference object with the holes of the other object by doing a non cyclic comparison, in order to allow that cases as figure 9 are considered as not equal as it is the case, because following a cyclic comparison for the holes they will be classified as equals. If all the holes in the reference object have a matching hole in the second object both objects are equal.

Figure 9. Two different objects with identical holes in different positions.

Applications

The theory here presented has been applied inside an application whose main objective is the automatic and intelligent recognition of mosaic tiles to be matched against a border design, in order to be able to assembly them automatically for creating mosaic borders of different designs. Later the reasoning process implemented will be used by a robot arm in the ceramic industry to assembly the mosaic tiles. A mosaic border is made from different tiles of different shapes, colours and sizes that once they are assembled they create a unique border with high added value (figure 10).

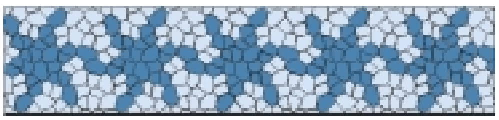


Figure 10. Example of a mosaic border design.

The theory has been implemented such that given as entry a file with the reference points and type of segment to which they belong, which will be extracted in a qualitative way from an image with different tiles (from now we call it as image) and a vectorial image of the design of the mosaic border (design), the application has to recognise which tile in the image appears in the design and match it against one representation of the tile in the design. Moreover, as the application has to allow a robot arm to place the tile in its correct place and orientation, and the tile can appear in a different orientation in the image and the design, then the angle of rotation to place the tile in the correct orientation according to the design is calculated. The angle of rotation δ is calculated using the mathematical concept of centroid explained above, as figure 11 shows graphically by following the steps detailed below.

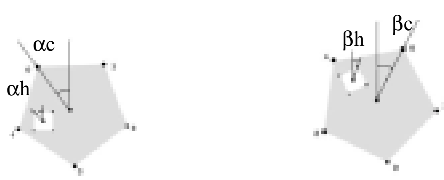


Figure 11. Angles for calculating the rotation angle

1. Find the vertex (vertex I) of the object in the design which corresponds to the upper-most left vertex of the tile in the image (vertex 0). If the object has a hole it is necessary to find the vertex I of the boundary of the container designed (vertex IC) with respect to the vertex 0 of the boundary of the container in the image, and the vertex I of the boundary of the hole (vertex IH) in the design with respect to the vertex 0 of the hole in the image.
2. Calculation of the angle α between the straight line following the direction vector along the x axes and crossing the centroid and the straight line crossing the vertex 0 of the tile and the centroid. If the object has holes it is necessary to calculate this angle α (called αc) for the container, and the angle α for the hole using the centroid of one of the holes (αh) and the vertex 0 of the hole selected (figure 11a).
3. Calculation of the same angles in the object in the design as it has be done in step 2, called βc (container) and βh (hole in the design corresponding to the one selected in 2) (figure 11b).
4. Calculus of the angle of rotation δ as:

If the object do not has a hole then:
if $(\beta-\alpha)>0$ *then* $\delta=(\beta-\alpha)$ *else* $\delta=(360+(\beta-\alpha))$
Else
If $(\beta h-\alpha h)>(\beta c-\alpha c)$ *then*
if $(\beta h-\alpha h)>0$ *then* $\delta=(\beta h-\alpha h)$ *else* $\delta=(360+(\beta h-\alpha h))$
//The angle is determined by the holes
Else
if $(\beta c-\alpha c)>0$ *then* $\delta=(\beta c-\alpha c)$ *else* $\delta=(360+(\beta c-\alpha c))$
//The angle is determined by the containers

Therefore, the tile in the image has to be rotated δ from its centroid to obtain the final orientation determined by design.

Moreover, as in this application the size of the objects is an important feature (for instance two squares of very different size are not the same piece), then the area of the shapes is considered. The area is needed too for the calculation of the centroid of the shapes, therefore we do not add more computational cost. The area once more is compared in a qualitative way. The limit to determine two tiles as the same is given by the joint (space leaved between two tiles when they are assembled). As the joint differs from one type of design to another it is given by the user of the application. Then if the difference between the areas of the tiles is less than the joint size the shapes represent the same object, otherwise they do not represent the same object.

Finally, for this application the qualitative model developed, including the colour and the size, is the one necessary and enough because there are not two exactly equal tiles.

Conclusions

A straightforward Qualitative Theory of Shape description is defined in this article. It will allow us to reason about shape in a qualitative way as human beings do. Most of the qualitative approaches developed up to now are used for reasoning about object position. The theory presented here allow us to use the same method to reason not only about position but also about shape. The theory proposed here provides a simple example for representing shapes without and with holes and with curves. The interest of this qualitative shape description relies in the fact that it is less constrained than metrical information but more constrained than topological information, which will not allow us to determine the convexity or concavity of the shape, neither the length of edges, nor the angle types.

The proposed theory has been applied to the recognition of tiles in a mosaic border design in order to allow the automatic and intelligent assembling of mosaic borders in the ceramic industry. This software will be applied in the future to a robot arm who places the tiles in the correct position to create the final ceramic mosaic strip designed. Moreover, for this application the theory presented gives a unique complete description of a shape.

Actually we are working in obtaining the file of the application which contains the reference points and the type of segment to which they belong. This file will be constructed in a qualitative way from an image taken by a vision system. Following a discrimination process only some points of the image are considered (therefore we reduce the computational cost). The granularity of the number of points considered varies in function of the design depending on the size of the curves. The qualitative slope associated to each point will be compared between points, and depending on the result of this comparison and using the bisection method the point is characterised as a reference point (a vertex, a point of maximum curvature or an inflection point, which will be a starting or ending point of a curve) or not.

An interesting future work will be the application of this theory to solve the problem of classification of objects, and then it could be used to the recognition and matching of objects partially occluded in the scene using the matching process for the visible part and the colour and texture of the objects.

Finally, another interesting application of the theory, that is being developed actually, is to apply it to determine the shape of the objects (obstacles) and the shape of the environment in which an autonomous robot is navigating. Taking into account this idea the theory has been described in order to work interactively with the reasoning process for robot navigation defined in (Felip & Escrig, 2003) and the concepts of the reasoning process defined in (Museros & Escrig, 2003).

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References

Biederman, I. (1987). Recognition-by-Components: A theory of Human Image Understanding. *Psychol. Rev., 94,* 2: 115-147.

Brady, M. (1983). Criteria for Representations of Shape. *Human and Machine Vision*.

Brisson, E. (1989). Representing Geometric Structures in d-dimensions: Topology and Order. *Proceedings 5th ACM Symposium on Computational Geometry*, Saarbruchen, 218-227.

Chen, S. And Freeman, H. (1990). Computing characteristic views of quadratic-surfaced solids. *Proc. 10th Int. Conf. On Pattern Recognition.*

Clementini, E. and Di Felice, P. (1997) A global framework for qualitative shape description. *GeoInformatica 1(1*):1-17.

Cohn, A.G. (1995). A Hierarchical Representation of Qualitative Shape based on Connection and Convexity. *Proceedings COSIT'95*, Springer-Verlag, 311-326.

Damski, J. C. and Gero, J. S. (1996). A logic-based framework for shape representation. *Computer-Aided Design 28(3)*:169-181.

Del Pobil, A.P., Serna, M.A. (1995). Spatial Representation and Motion Planning. *Lecture Notes in Computer Science no. 1014,* Springer-Verlag.

Escrig, M. T and Toledo, F. (1998). *Qualitative Spatial Reasoning: Theory and Practice*, IOS PRESS.

Felip, P., Escrig, M.T. (2003). Qualitative Spatial Reasoning for Robot Path Planning. *IOS-Press Frontiers in Artificial Intelligence and Applications vol. 100*: 99-110.

Ferrucci, V. and Paoluzzi, A. (1991). Extrusion and Boundary Evaluation for Multidimensional Polyhedra. *Computer Aided Design,* 23, 1: 40-50.

Flynn , P.J. and Jain, A.K. (1991). CAD-based Computer Vision: From CAD Models to Relational Graphs. *IEEE Transaction on P.A.M.I., 13:*114-132.

Frank, A.U. (1991). Qualitative Spatial Reasoning with Cardinal Directions. *Proceedings of the Seventh Austrian Conference on Artificial Intelligence*, Springer, pp. 157-167.

Freeman, H. And Chakravarty, I. (1980). The use of characteristic views in the recognition of three-dimensional objects. *Pattern Recognition in Practice.*

Hernández, D., Clementini, E. and Di Felice, P. (1995) Qualitative Distances. *Frank and Kuhn (eds.) Spatial Information Theory. A theoretical Basis for GIS. International Conference COSIT'95, volume 988 of Lecture Notes in Computer Science*, Springer.

Isli, A., Museros, Ll., Barkowsky T. and Reinhard M. A. (2000). Topological Calculus for Cartographic Entities. *Lecture notes in Artificial Intelligence 1849. Spatial Cognition II. Integrating Abstract Theories, Empirical Studies, Formal Methods and Practical Applications. Spatial Cognition.* Springer-Verlag, pp. 225 – 239.

Jungert, E. (1994). Symbolic spatial reasoning on object shapes for qualitative matching. *Spatial Information Theory*. Springer Verlag, Berlin, 444-462.

Leyton, M., (1988). A process-grammar for shape. *Artificial Intelligence, 34*:213-247.

Museros L., Escrig M.T. (2003). "Modeling Motion by the Integration of Topology and Time". *Journal of Universal Computer Science. September Issue. Vol. 9*, No. 9.

Park, S.H. and Gero, J. S. (1999). Qualitative representation and reasoning about shapes. *Gero, J. S. and Tversky, B. (eds), Visual and Spatial Reasoning in Design* , pp. 55-68.

Randell, D.A., Cui, Z. And Cohn, A.G. (1992). A spatial logic based on regions and connection. *Proceedings 3rd Int. Conf. On Knowledge Representation and Reasoning,* Morgan Kaufmann.

Requicha, A.A.G. (1980). Representations of Rigid Solids: Theory, Methods, and Systems, *ACM Computing Surveys, 12, 4*:437-464.

Schlieder, Ch. (1996). Qualitative Shape Representation. *Spatial conceptual models for geographic objects with undertermined boundaries*, Taylor and Francis, London, 123-140.

Steger, C. (1996). *On the Calculation of Arbitrary Moments of Polygons* (Tech. Rep. FGBV-96-05). Technische Universität München.