Qualitative Modelling of Kinematic Robots

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Abstract

This study presents an approach, the unit circle (UC), to the qualitative representation of kinematic robots. A robot is described as a collection of constraints holding among time-varying, interval-valued parameters. The UC representation is presented, and the continuous motion of the end-effector is evaluated by the change of directions of qualitative angle and qualitative length. Analytical formulas of qualitative velocity and qualitative acceleration are derived. The characteristic mapping is introduced for fault detection and diagnosis in terms of the UC. In the end simulation results demonstrate the effectiveness of the UC approach for fault diagnosis. The UC representation of robots concerns a global assessment of the systems behaviour, and it might be used for the purpose of monitoring, diagnosis, and explanation of physical systems. This is the first step to fault diagnosis and remediation for Beagle 2 using qualitative methods.

1. Introduction

Qualitative and quantitative methods are two ways of looking at the world and solving problems. They have their advantages and disadvantages when used as solutions to particular problems. When the tradeoff between computational complexity and accuracy is a major problem, qualitative reasoning methods are usually considered as the preferred solutions. The problem domain related to monitoring, diagnosis and explanation can be easily resolved by taking advantage of qualitative reasoning methods rather than quantitative methods. This paper proposes a novel qualitative modelling scheme for the representation of planar robots, the unit circle (UC). The approach can be easily extended to spatial robots. This is the first attempt to clearly define the qualitative representation of robots, whose end-effector's position can be described by a qualitative length and a qualitative orientation angle within a unit circle. Qualitative analysis of a robot is constructed in terms of subsets of a unit circle with link sequence constraints. The UC approach also derives qualitative velocity and qualitative acceleration based on qualitative length and orientation angle. The characteristic mapping presents the mapping relation between inputs and outputs of a physical system using characteristic values, which are characteristic quantities extracted from quantitative intervals of a domain to describe their corresponding qualitative information. The selection of characteristic values is application-dependent, it is determined by the mean value, the minimum and the maximum of quantitative intervals in this paper. The characteristic mapping basically provides approximate solutions to that may be used to guide the application of quantitative methods.

The paper is organized as follows, the related work is given in Section 2. The UC qualitative representation of planar robots is presented in Section 3. The characteristic mapping is addressed in Section 4. A case study is given in Section 5 to prove the proposed approach; discussions and conclusion are drawn at the end.

2. Related Work

Qualitative kinematics is a branch of qualitative mechanics concerned with motion in qualitative space without reference to force or mass. Qualitative modelling did not really offer a unified successful framework, primarily due to the complexity of 3-D space problems and secondly due to the comlexity when translational joints and rotational joints are mixed. However there does exist a huge growing interest in qualitative spatial reasoning, qualitative physics and even cognitive science. They have made contributions to qualitative representation of geometry, which is the basis for qualitative kinematics of physical systems.

In the area of qualitative analysis of physical systems a number of approaches have been developed. Artificial intelligence methods for qualitative reasoning about mechanisms were first developed by Rieger and Grinberg, whose system produces realistic qualitative simulations of the behaviour of mechanisms based on their knowledge representation consisting of events, tendencies, states, and state changes, related by several different types of causal links. McDermott created an extended representation on a better logical foundation that is capable of addressing a larger set of issues, based on the knowledge representation of Rieger and Grinberg. Nielsen described a theory of qualitative mechanics, the symbolic analysis of the motions and the geometric interaction of physical objects, for analysis of rigid body mechanisms. The most significant work on qualitative mechanism analysis is that of Faltings. He built upon his and Forbus' earlier work on qualitative kinematics, and developed a first principles algorithm for analyzing planar mechanisms. He introduced a "theory of place vocabulary" which formed the basis for an envisionment of the qualitative behaviour of a device under external influences. However, this work suffered from the limitation that certain problems could not be solved without including quantitative information. Therefore, in contrast to the configuration space approach of Faltings, Olivier et al have proposed a qualitative kinematics reasoning method based upon the use of occupancy arrays (Oliver *et al*). This approach does not require inference rules. It works simply on the constraint that no two objects occupy the same occupancy array position, and can be extended to including semi-quantitative information.

Kramer developed 'The Linkage Assistant' kinematic simulator which demonstrated that mechanism kinematic analysis did not solely have to rely on exact geometric mechanism information, i.e. a qualitative approach could be adopted. Liu presented a qualitative representation and reasoning approach based upon the formalism of qualitative trigonometry, qualitative arithmetic, and qualitative spatial inferencing. The formalism has been applied successfully to both closed-chain constrained, and open-chain underconstrained, 2D multiple linkage problems. Stahovich *et al* presented a theory of qualitative rigid-body mechanics to demonstrate a program, *SKETCHIT*, that uses this theory to compute qualitative rigid-body dynamic simulation. *SKETCHIT* can handle devices that are composed of an arbitrary number of fixed-axis components and springs, with driving inputs coming from both applied motions and forces.

Engineering design, like robotic navigation, ultimately normally requires a fully metric description. However, at the early stages of the design process, a reasonable qualitative description suffices. The field of qualitative kinematics is largely concerned with supporting this type of activity (Forbus *et al*).

3. Qualitative Position Representation of Planar Robots

In this section the UC approach is proposed for qualitative analysis of planar robots. An n-link serial robot, combined by links and joints, can be decomposed into n link-based segments, each of which consists of one link and its corresponding joint. Each segment can be described by a qualitative length and a qualitative orientation angle, furthermore the qualitative representation of the end-effector of the robot is provided by the qualitative information of each link segment. With respect to the n-link robot, the direct kinematics of the robot can be described with the following equations.

$$\begin{aligned} x &= p(\theta) \\ \dot{x} &= \mathbf{J}(\theta)\dot{\theta} \\ \ddot{x} &= \mathbf{J}(\theta)\ddot{\theta} + \dot{\mathbf{J}}\left(\theta,\dot{\theta}\right)\dot{\theta} \end{aligned} \tag{1}$$

where

$$p(\theta) = \begin{bmatrix} p_x(\theta) \\ p_y(\theta) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} l_i \cos\theta_i \\ \sum_{i=1}^{n} l_i \sin\theta_i \end{bmatrix}$$

Where p is an n-dimensional vector function representing direct kinematics. Compared with quantitative methods, the description of the ith link segment in qualitative reasoning requires qualitative position parameters, qp^i, that is, the qualitative length of the ith link, qp_l^i, and the qualitative angle, qp_θ^i. Then the qualitative description is given in terms of constraint information.

$$qp^i = \begin{cases} qp_l^i \left| qp_l^i \in [0, l_i] \right. \\ qp_\theta^i \left| qp_\theta^i \in [0, 2\pi] \right. \end{cases} \tag{2}$$

Further, the intervals of the length and the orientation angle of the ith link segment are described by length parameter r_i and orientation parameter s_i. The setting of the two parameters is subject to system requirements such as joint offset and so forth. The value of s_i and r_i are application-dependent.

$$\begin{cases} qp_l^i \left| qp_l^i \in \left[0, l_{i1}, l_{i2}, \cdots, l_{i(r_i-1)}, l_{ir_i}\right] \right. \\ qp_\theta^i \left| qp_\theta^i \in \left[0, q\theta_{i1}, q\theta_{i2}, \cdots, q\theta_{i(s_i-2)}, q\theta_{i(s_i-1)}, 2\pi\right] \right. \end{cases} \tag{3}$$

where

$$\begin{aligned} 0 \le l_{i1} < l_{i2} < \cdots < l_{i(r_i-1)} < l_{ir_i} \le l_i \\ 0 \le q\theta_{i1} < q\theta_{i2} < \cdots < q\theta_{i(s_i-1)} \le 2\pi \end{aligned}$$

The length intervals and the orientation intervals of the UC approach are used to meet the system requirements regarding qualitative position and qualitative orientation. The comparison between the desired data and the actual data

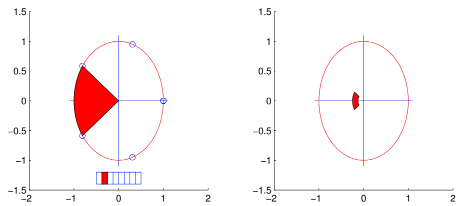

Figure 1. Separated and integral UC representation

in the UC representation can provide sufficient evidence for fault detection and diagnosis. For example, the link before deformation can be shorter, equal, longer in comparison with the link after deformation, meanwhile some of the domain can be defined as a bending-safe area, the others as bending-damaged areas for diagnostic purposes in link bending analysis. The separate description of qualitative information of the position and the orientation can be integrated into a unit circle; the active area is highlighted as shown in Fig. 1.

3.1. Qualitative position of unit circle representation

For a global assessment of a system behaviour, the functional qualitative constraint, $(Y = M^{+}(X))$, is applied so that the interval-valued parameters of the ith link segment are replaced by the proportion of the interval-valued parameters of the ith link segment to the addition of the lengths of all link segments. It is noted that the link segments are connected by the link sequence constraints, $l_1, l_2, \cdots, l_i, \cdots, l_n$, from which, the UC qualitative representation of the position of the end-effector of the n-link robot can be derived.

$$\begin{cases} qp_l = \bigoplus_{i=1}^{n} qp_l^i \left| qp_l^i \in UC_{ql} \times \sum_{i=1}^{n} l_i \right. \\ qp_\theta = \bigoplus_{i=1}^{n} qp_\theta^i \left| qp_\theta^i \in UC_{q\theta_i} \times 2\pi \right. \end{cases} \tag{4}$$

where

$$\begin{cases} UC_{qp_l} = \left[0, \frac{l_{11}}{\sum_{i=1}^{n} l_i}, \frac{l_{12}}{\sum_{i=1}^{n} l_i}, \cdots, \frac{l_{i1}}{\sum_{i=1}^{n} l_i}, \cdots, \frac{l_{n(r_n-1)}}{\sum_{i=1}^{n} l_i}, \frac{\sum_{i=1}^{n} l_i}{\sum_{i=1}^{n} l_i}\right] \\ UC_{qp_\theta^i} = \left[0, \frac{q\theta_{i1}}{2\pi}, \frac{q\theta_{i2}}{2\pi}, \cdots, \frac{q\theta_{i(s_i-1)}}{2\pi}, \frac{2\pi}{2\pi}\right] \end{cases}$$

Here UC_{qp_l} stands for the qualitative length of the ith link segment of a unit circle, $UC_{qp_\theta^i}$ for the qualitative orientation angle of the unit circle.

Definition 1: The qualitative position of a point in space can be described by a pair of qualitative parameters such as a qualitative length and a qualitative orientation relative to a fixed reference coordinate system.

Definition 2: The qualitative position of the end-effector of a robot can be described by a pair of qualitative position and qualitative orientation in a unit circle, which are provided by combination of qualitative parameters of all segment links.

Each interval-valued area of the unit circle, $UC_{ql}, UC_{q\theta_i}$ corresponds to the qualitative meaning in domain knowledge representation. The representation conversion of a particular position 'Q' of the end-effector of a n-link serial robot from quantitative to qualitative description is given in Fig.2. The robot is described in terms of Cartesian coordinates; its qualitative representation of the UC is in terms of the qualitative angle and the qualitative length of the end-effector. It is defined by a qualitative vector $\vec{Q'}$ from the origin to the position Q. For example, the qualitative orientation can be divided into $front$, $back$, $left$ and $right$. The qualitative length can be identified by $less$, $equal$ and $larger$ three regions in qualitative description.

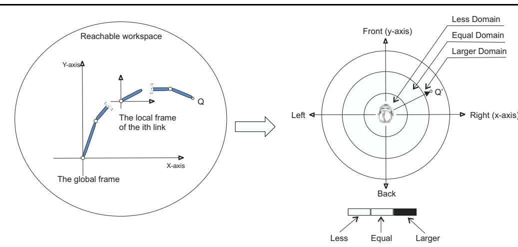

Figure 2. The representation conversion from quantitative viewpoint to qualitative

In position mapping from the qualitative representation to continuous spatial quantities, one of standard assumptions in traditional qualitative reasoning is that change is continuous. That is, in addition to qualitative magnitude such as qualitative angles and qualitative lengths in the quantity space, we need to know the direction of change of each variable. Thus for each variable, we describe its qualitative state in terms of its magnitude in the quantity space and its direction of change: increasing, decreasing or steady.

$$\left[\Delta qp^k\right] = sign\left(qp^{k+1} - qp^k\right) = \begin{cases} + & \Delta qp^k > 0 \\ - & \Delta qp^k < 0 \\ 0 & \Delta qp^k = 0 \end{cases}$$

Where qp_i^k denotes the qualitative position parameters within the kth interval. Then the qualitative positions of the end-effector of a robot can be described as follows,

$$\left[\Delta qp^{k}\right] = \bigoplus_{i=1}^{k} \left[\sum_{i=1}^{n} \Delta qp_{l_i}^{k}\right] \bigoplus_{i=1}^{k} \left[\sum_{i=1}^{n} \Delta qp_{\theta_i}^{k}\right]$$

For robotic qualitative kinematics, the continuous motion of robots can be described by the combination of magnitudes of qualitative parameters with their direction of change. The direction of change of a qualitative orientation angle is defined by a qualitative orientation vector, whose direction in perpendicular to the corresponding qualitative vector; the direction of change of a qualitative length is defined by a qualitative length vector, whose direction is vertical to the qualitative vector. The anticlockwise direction of qualitative orientation angles is denoted as positive, and the direction of facing the origin of qualitative lengths as positive. See Fig.3.

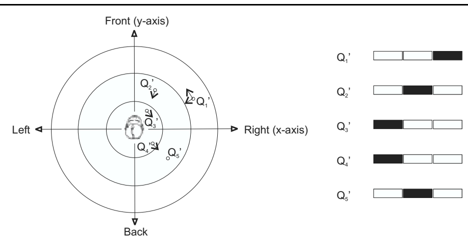

Figure 3. The mapping definition from qualitative representation to continuous spatial quantities

The continuous motion, from the position $Q_1^{'}$ to $Q_5^{'}$, of an end-effector is given in Fig 3, the qualitative description is in Table 1, the direction of change can be decided by the qualitative magnitudes of continuous motion states, or predefined by robotic planners.

	$[q\theta]$	$[ql]$	$[\Delta q\theta]$	$[\Delta ql]$
$Q_1^{'}$	$FrontRight$	$Large$	$Increasing$	$Decreasing$
$Q_2^{'}$	$FrontRight$	$Equal$	$Steady$	$Decreasing$
$Q_3^{'}$	$FrontRight$	$Small$	$Decreasing$	$Steady$
$Q_4^{'}$	$BackRight$	$Smalll$	$Steady(0)$	$Increasing$
$Q_5^{'}$	$BackRight$	$Equal$	$Steady$	$Steady$

Table 1. The qualitative description of a continuous motion

3.2. Qualitative velocity representation

Qualitative velocities can be used for describing the rate of change of qualitative positions.

Definition 3: The qualitative velocity of a point in space is the derivative of the qualitative positions of the point relative to any given reference system,

$$qv = \frac{dqp}{dt} \approx \frac{\Delta qp}{\Delta t} \tag{5}$$

Definition 4: The qualitative velocity of the end-effector of a robot, qv, consisting of qualitative linear velocity, qv_l, and qualitative angular velocity, qv_θ, is the derivative of the qualitative position of a state such as qualitative length, qp_l, and qualitative orientation, qp_θ. For the robotic velocity of a robot, we have the following:

$$\begin{aligned} qv_l &= \frac{dqp_l}{dt} \approx \frac{\Delta qp_l}{\Delta t} \\ qv_\theta &= \frac{dqp_\theta}{dt} \approx \frac{\Delta qp_\theta}{\Delta t} \end{aligned} \tag{6}$$

As the relationship between the certainty values of particular values is characterised by the partial derivative.

$$\begin{aligned} \Delta qv_l &= \frac{\partial qv_l}{\partial qp_l} \Delta qp_l \\ \Delta qv_\theta &= \frac{\partial qv_\theta}{\partial qp_\theta} \Delta qp_\theta \end{aligned}$$

Assuming an initial state of the end-effector of the robot is qp_{l0}, $qp_{\theta 0}$, qv_{l0}, $qv_{\theta 0}$, and then we have the following in terms of the mean value theorem of differentiation,

$$\begin{aligned} &\Delta qv_l = qv_l - qv_{l0} = \frac{\partial qv_l}{\partial qp_l} \Delta qp_l = \frac{\partial qv_l}{\partial qp_l} \left(qp_l - qp_{l0}\right) \\ &qv_l = \frac{\partial qv_l}{\partial qp_l} \left(qp_l - qp_{l0}\right) + qv_{l0} \end{aligned}$$

The interval-based value of qp_l can be substituted from the Eq. (6). The direction of the velocity can be calculated from the following,

$$\begin{aligned} \left[\Delta qv_l\right] &= \left[\frac{\partial qv_l}{\partial qp_l}\right] \left[\Delta qp_l\right] \\ \left[\Delta qv_\theta\right] &= \left[\frac{\partial qv_\theta}{\partial qp_\theta}\right] \left[\Delta qp_\theta\right] \end{aligned}$$

Further, the qualitative description of general velocity is derived.

$$\begin{aligned} \left[\Delta qv\right] &= \bigoplus_{i=1}^{n} \left[\sum_{i=1}^{n} \Delta qv_{l_i}\right] \bigoplus_{i=1}^{n} \left[\sum_{i=1}^{n} \Delta qv_{\theta_i}\right] \\ &= \bigoplus_{i=1}^{n} \left[\sum_{i=1}^{n} \frac{\partial qv_{l_i}}{\partial qp_{l_i}} \Delta qp_{l_i}\right] \bigoplus_{i=1}^{n} \left[\sum_{i=1}^{n} \frac{\partial qv_{\theta_i}}{\partial qp_{\theta_i}} \Delta qp_{\theta_i}\right] \end{aligned} \tag{7}$$

3.3. Qualitative acceleration representation

The representation of qualitative acceleration is presented in this section.

Definition 5: The qualitative acceleration of a point in space is the double derivative of the positions of the state,

$$qa = \frac{dqv}{dt} = \frac{d^2ql}{d^2t} \approx \frac{\Delta qv}{\Delta t} \approx \frac{\Delta ql}{\Delta^2 t}$$

Definition 6: The qualitative acceleration of the end-effector of a robot is the derivative of the velocities of the state, or the double derivative of the state. Firstly we have,

$$\begin{aligned} qa_l &= \frac{dqv_l}{dt} = \frac{d^2qp_l}{d^2t} \approx \frac{\Delta qv_l}{\Delta t} \approx \frac{\Delta qp_l}{\Delta^2 t} \\ qa_\theta &= \frac{dqv_\theta}{dt} = \frac{d^2qp_\theta}{d^2t} \approx \frac{\Delta qv_\theta}{\Delta t} \approx \frac{\Delta qp_\theta}{\Delta^2 t} \end{aligned}$$

Then,

$$\begin{aligned} [\Delta qa_l] &= \frac{\partial qa_l}{\partial qv_l}[\Delta qv_l] = \frac{\partial qa_l}{\partial qp_l}[\Delta qp_l] \\ [\Delta qa_\theta] &= \frac{\partial qa_\theta}{\partial qv_\theta}[\Delta qv_\theta] = \frac{\partial qa_\theta}{\partial qp_\theta}[\Delta qp_\theta] \end{aligned}$$

where $[\Delta qa] = sign(qa_{k+1} - qa_k)$. Finally, the qualitative description of general acceleration is derived,

$$\begin{aligned} [\Delta qa] &= \bigoplus_{i=1}^{n}\left[\sum_{i=1}^{n}\Delta qa_{l_i}\right]\bigoplus_{i=1}^{n}\left[\sum_{i=1}^{n}\Delta qa_{\theta_i}\right] \\ &= \bigoplus_{i=1}^{n}\left[\sum_{i=1}^{n}\frac{\partial qa_{l_i}}{\partial qv_{l_i}}\Delta qv_{l_i}\right]\bigoplus_{i=1}^{n}\left[\sum_{i=1}^{n}\frac{\partial qa_{\theta_i}}{\partial qv_{\theta_i}}\Delta qv_{\theta_i}\right] \\ &= \bigoplus_{i=1}^{n}\left[\sum_{i=1}^{n}\frac{\partial qa_{l_i}}{\partial qp_{l_i}}\Delta qp_{l_i}\right]\bigoplus_{i=1}^{n}\left[\sum_{i=1}^{n}\frac{\partial qa_{\theta_i}}{\partial qp_{\theta_i}}\Delta qp_{\theta_i}\right] \end{aligned} \tag{8}$$

4. The Characteristic Mapping

The goal of qualitative reasoning is to provide approximate solutions that may be used to guide the application of quantitative methods. The characteristic mapping basically extracts characteristic quantities from quantitative interval to describe the corresponding qualitative information. Stability of the characteristic mapping is obvious for linear systems; for nonlinear systems, Kawamura and Shima [10] proved the robust stability with the condition that real and imaginary part of their characteristic polynomial $F(s)$ are monotonic parameters in the frequency domain. The methods used to select characteristic values are application dependent such as landmark methods.

For robotic fault diagnosis, let $f\left(\theta_t^d\right)$ be the desired trajectory of the position of the end-effector, $f\left(\theta_t^a\right)$ that from the sensors and the corresponding error $\Delta f(t)$. The following formula differentiates faulty and non-faulty intervals.

$$|\Delta f(t)| \le \varepsilon \tag{9}$$

where ε is the fault index determined by system requirement such as joint offsets.

The characteristic quantities of each fault interval, $\hat{\theta}$s, are determined by the time instants, t_{lmin}, t_{lmax}, where the local maximum and minimum of $\Delta f(t)$ are achieved. Note that the selection of the intervals is application-dependent in order to make sure that suitable local characteristic values are chosen.

So far the characteristic values of each faulty interval are extracted, which can describe the input qualitative states of the robot in each faulty interval. These include fault and noise signals as well if the inputs are from measurement. The output qualitative states, the positions of robot end-effector, can be calculated based on robotics. The position of the end-effector of a robot can be calculated in terms of equation (1) as,

$$\begin{aligned} \hat{\alpha_p} &= \tan^{-1}\left(\frac{p_y\left(\hat{\theta}_i\right)}{p_x\left(\hat{\theta}_i\right)}\right) = \tan^{-1}\left(\frac{\sum_{i=1}^{n} l_i \sin\hat{\theta}_i}{\sum_{i=1}^{n} l_i \cos\hat{\theta}_i}\right) \\ \hat{r_p} &= \sqrt{\left(\sum_{i=1}^{n} l_i \sin\hat{\theta}_i\right)^2 + \left(\sum_{i=1}^{n} l_i \cos\hat{\theta}_i\right)^2} \end{aligned} \tag{10}$$

Where $i = t_{lmax}\ t_{lmin}$. Thus, the qualitative information of each interval are $qp_\theta = [\hat{\alpha_p}]$ and $qp_l = [\hat{r_p}]$. Hence, the kth state position can be defined by two characteristic values

$$\begin{aligned} SP_{k,1} &= \left(qp_\theta\left(\left[\alpha_p^{2k-1}\right]\right), qp_l\left(\left[r_p^{2k-1}\right]\right)\right) \\ SP_{k,2} &= \left(qp_\theta\left(\left[\alpha_p^{2k}\right]\right), qp_l\left(\left[r_p^{2k}\right]\right)\right) \end{aligned}$$

The state change of continuous motion in the tth state position,

$$\Delta SC_k = sign\left(SP_{k,2} - SP_{k,1}\right) = \bigoplus_{i=1}^{n}\left[\Delta qp_\theta^k\right]\bigoplus_{i=1}^{n}\left[\Delta qp_l^k\right]$$

where $\begin{aligned} \left[\Delta qp_\theta^k\right] &= sign\left(qp_\theta\left(\left[\alpha_p^{2k}\right]\right) - qp_\theta\left(\left[\alpha_p^{2k-1}\right]\right)\right) \\ \left[\Delta qp_l^k\right] &= sign\left(qp_l\left(\left[r_p^{2k}\right]\right) - qp_l\left(\left[r_p^{2k-1}\right]\right)\right) \end{aligned}$

5. Case study

A case study of the UC representation of the simplified robot arm of Beagle 2 Lander in Figure 4 is addressed in this section.

$$P(\Theta) = \begin{bmatrix} p_x(\theta_1,\theta_2,\theta_3) \\ p_y(\theta_1,\theta_2,\theta_3) \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{3}\left(l_k \cos\left(\sum_{i=1}^{k}\theta_i\right)\right) \\ \sum_{k=1}^{3}\left(l_k \sin\left(\sum_{i=1}^{k}\theta_i\right)\right) \end{bmatrix}$$

Where l_1, l_2, l_3, are link lengths, and θ_1, θ_2, θ_3, are orientation angles, respectively. The UC representation of the end-effector's trajectory is tested based on orientation angles without fault and with fault.

5.1. Parameters setting

The setting of length and orientation parameters of the UC representation is application-dependent. The UC can be applied to general dynamic systems including process control systems and physical systems, whose system requirements can be met by functions of the UC toolbox developed in $Matlab/Simulink^{TM}$. In this paper, the parameter r is set as 19 qualitative units due to that the maximum offset of the end-effector position is given by $0.08m$; the parameter of s as 20 qualitative units because two characteristic values are chosen in the interval of 0.25 second. The three joint trajectories of the robot, θ_1, θ_2, θ_3, have been given in Figure 5. The large elliptic trajectories are those of the end-effector.

5.2. Fault detection

The first principle for the fault detection of robots is that no fault occurs if and only if the actual qualitative states remain within the coverage of the UC representation of the desired trajectories. The comparison of the two UC representation versions is demonstrated in Figure 7, the fault of the end-effector is clearly identified by the four deep dark segments, which is generated by characteristic mappings from fault joint trajectories. The fault area in the UC representation describes correspondingly the dash-line fish-shaped fault trajectory of the end-effector. Hence the fault global assessment is sufficiently reconstructed using the UC fault version.

5.3. Fault isolation

How to locate fault in terms of fault detection is another issue of fault diagnosis. Fault isolation has been made a lot easier by isolating functions of the UC. The inference carries out based on characteristic value of fault segments in the UC. The faults can be classified into three types, single faults, and multiple faults not happening in the same interval of the UC, multiple faults whose characteristic values are not in the same time instant. For single fault herein, the analysis is shown in table 2, In which [DT], [AT] denote the desired and actual qualitative values of joint trajectories as shown and 6. From the table 2, the faulty link segment 1 is detected by the fact that the actual results of joint 1 are equal to the corresponding actual results of the faulty characteristic mapping in terms of the desired inputs of joint 2 and 3 rather than the desired results of joint 1. The same analysis is effective for the other two types of faults. Further research will examine multiple faults whose characteristic values exist on the same-time point, even though the possibility of those faults happening is very small. The case study proved that the UC representation can not only generate qualitative models for robotic systems, but also detect and isolate faults. Though the proposed qualitative technique is presented in terms of robotics, it can be extended to general machines by adoption of qualitative description of the orientation and translation component of a robot.

$Joint\ 1$	$Joint\ 2$	$Joint\ 3$	$fault\ location$
$=[AT]$ $\neq[DT]$	$[DT]$	$[DT]$	$linksegment\ 1$
$[DT]$	$[DT]$	$\neq[AT]$ $\neq[DT]$	$nofault$
$[DT]$	$\neq[AT]$ $\neq[DT]$	$[DT]$	$nofault$

Table 2. Fault isolation analysis based on characteristic mapping

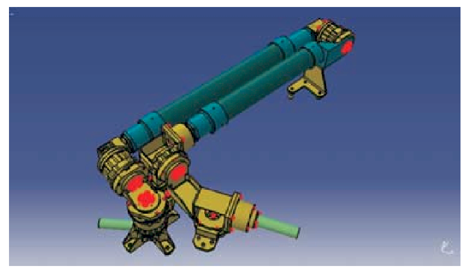

Figure 4. the simplified robot arm of Beagle 2 Lander

6. Conclusion

In this paper a novel qualitative modelling of kinematic robots has been proposed. The unit circle approach is addressed to bridge quantitative data and qualitative state for qualitative modelling. The position and orientation properties of the end-effectors of robots and their link segments are derived and analytical formulas of qualitative velocity and qualitative acceleration is derived based on qualitative position information. Characteristic mapping is introduced to transfer qualitative information between system inputs and outputs in terms of characteristic values of the UC intervals. Finally the case study of a three link robot demonstrated the feasibility of the UC approach, which can deal with faults at the link segment level.

Though the proposed qualitative technique is presented in terms of robotics, it can be extended to general machines.

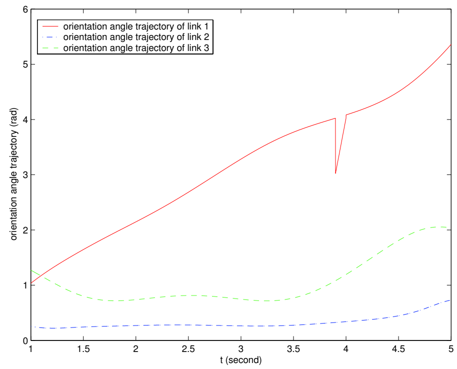

Figure 5. the known input parameters of orientation angles, θ_1, θ_2, θ_3, of which, θ_1 carries fault signal

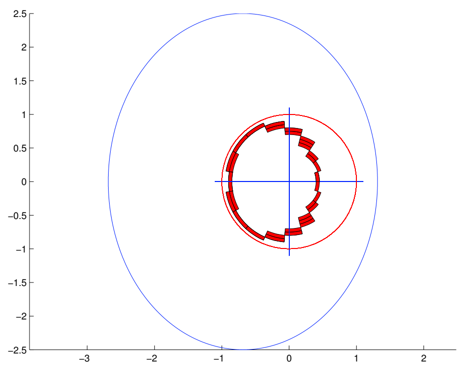

Figure 6. the UC representation with trajectory of end-effector of the robot ($r = 19$, $s = 20$)

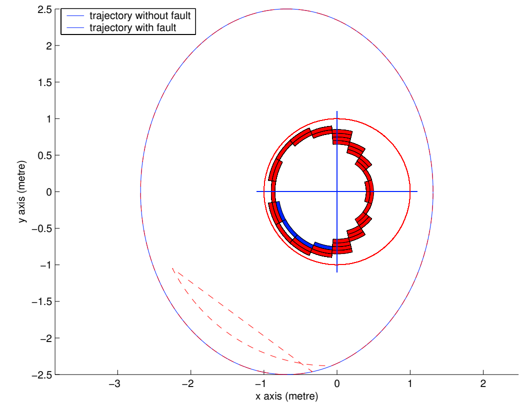

Figure 7. Comparison of UC representations ($r = 19$, $s = 20$) of trajectory of end-effector of the robot with both signals from Figure 5

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